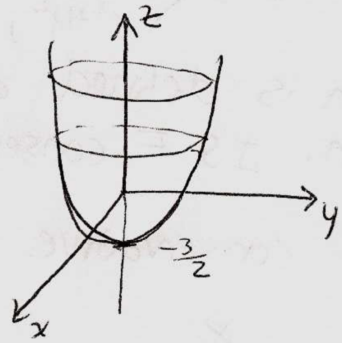
③ Find the minimum distance from a point on the surface

$$z = x^2 + y^2 - 3/2$$

to the origin.



- minimize $(x-0)^2 + (y-0)^2 + (z-0)^2$

subject to constraint $g(x,y,z) = x^2+y^2-z = \frac{3}{2}$

$$\nabla f = \lambda \nabla g$$

$$\langle 2x, 2y, 2z \rangle = \lambda \langle 2x, 2y, -1 \rangle$$

$2x = \lambda 2x \rightarrow \lambda = 1$ or $x = 0$

$2y = \lambda 2y \rightarrow \lambda = 1$ or $x = 0$

$2z = -\lambda$

$x^2+y^2-z = \frac{3}{2}$

if $\lambda = 1$: $z = -\frac{1}{2}$ so $x^2+y^2 = 1$ ← (intersection of paraboloid with plane $z = -\frac{1}{2}$, disk radius 1)

if $x=0$ and $y=0$ $z = -\frac{3}{2}$

distance $= \sqrt{(-\frac{3}{2})^2} = \frac{3}{2}$ for $(0,0,-\frac{3}{2})$

distance $= \sqrt{x^2+y^2+z^2} = \sqrt{1+(-\frac{1}{2})^2} = \sqrt{\frac{5}{4}}$ for disk $x^2+y^2=1$

since $\frac{3}{2} > \sqrt{\frac{5}{4}}$, then $\sqrt{\frac{5}{4}}$ is the minimum distance.